
CHAPTER 8

INTEREST RATES AND BOND VALUATION

Answers to Concept Questions

1. No. As interest rates fluctuate, the value of a Treasury security will fluctuate. Long-term Treasury securities have substantial interest rate risk.
2. All else the same, the Treasury security will have lower coupons because of its lower default risk, so it will have greater interest rate risk.
3. No. If the bid were higher than the ask, the implication would be that a dealer was willing to sell a bond and immediately buy it back at a higher price. How many such transactions would you like to do?
4. Prices and yields move in opposite directions. Since the bid price must be lower, the bid yield must be higher.
5. Bond issuers look at outstanding bonds of similar maturity and risk. The yields on such bonds are used to establish the coupon rate necessary for a particular issue to initially sell for par value. Bond issuers also simply ask potential purchasers what coupon rate would be necessary to attract them. The coupon rate is fixed and simply determines what the bond's coupon payments will be. The required return is what investors actually demand on the issue, and it will fluctuate through time. The coupon rate and required return are equal only if the bond sells for exactly at par.
6. Yes. Some investors have obligations that are denominated in dollars; i.e., they are nominal. Their primary concern is that an investment provides the needed nominal dollar amounts. Pension funds, for example, often must plan for pension payments many years in the future. If those payments are fixed in dollar terms, then it is the nominal return on an investment that is important.
7. Companies pay to have their bonds rated simply because unrated bonds can be difficult to sell; many large investors are prohibited from investing in unrated issues.
8. Treasury bonds have no credit risk since it is backed by the U.S. government, so a rating is not necessary. Junk bonds often are not rated because there would be no point in an issuer paying a rating agency to assign its bonds a low rating (it's like paying someone to kick you!).
9. The term structure is based on pure discount bonds. The yield curve is based on coupon-bearing issues.
10. Bond ratings have a subjective factor to them. Split ratings reflect a difference of opinion among credit agencies.

11. As a general constitutional principle, the federal government cannot tax the states without their consent if doing so would interfere with state government functions. At one time, this principle was thought to provide for the tax-exempt status of municipal interest payments. However, modern court rulings make it clear that Congress can revoke the municipal exemption, so the only basis now appears to be historical precedent. The fact that the states and the federal government do not tax each other's securities is referred to as "reciprocal immunity."
12. Lack of transparency means that a buyer or seller can't see recent transactions, so it is much harder to determine what the best bid and ask prices are at any point in time.
13. When the bonds are initially issued, the coupon rate is set at auction so that the bonds sell at par value. The wide range of coupon rates shows the interest rate when the bond was issued. Notice that interest rates have evidently declined. Why?
14. Companies charge that bond rating agencies are pressuring them to pay for bond ratings. When a company pays for a rating, it has the opportunity to make its case for a particular rating. With an unsolicited rating, the company has no input.
15. A 100-year bond looks like a share of preferred stock. In particular, it is a loan with a life that almost certainly exceeds the life of the lender, assuming that the lender is an individual. With a junk bond, the credit risk can be so high that the borrower is almost certain to default, meaning that the creditors are very likely to end up as part owners of the business. In both cases, the "equity in disguise" has a significant tax advantage.
16.
 - a. The bond price is the present value of the cash flows from a bond. The YTM is the interest rate used in valuing the cash flows from a bond.
 - b. If the coupon rate is higher than the required return on a bond, the bond will sell at a premium, since it provides periodic income in the form of coupon payments in excess of that required by investors on other similar bonds. If the coupon rate is lower than the required return on a bond, the bond will sell at a discount since it provides insufficient coupon payments compared to that required by investors on other similar bonds. For premium bonds, the coupon rate exceeds the YTM; for discount bonds, the YTM exceeds the coupon rate, and for bonds selling at par, the YTM is equal to the coupon rate.
 - c. Current yield is defined as the annual coupon payment divided by the current bond price. For premium bonds, the current yield exceeds the YTM, for discount bonds the current yield is less than the YTM, and for bonds selling at par value, the current yield is equal to the YTM. In all cases, the current yield plus the expected one-period capital gains yield of the bond must be equal to the required return.
17. A long-term bond has more interest rate risk compared to a short-term bond, all else the same. A low coupon bond has more interest rate risk than a high coupon bond, all else the same. When comparing a high coupon, long-term bond to a low coupon, short-term bond, we are unsure which has more interest rate risk. Generally, the maturity of a bond is a more important determinant of the interest rate risk, so the long-term, high coupon bond probably has more interest rate risk. The exception would be if the maturities are close, and the coupon rates are vastly different.

Solutions to Questions and Problems

NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

NOTE: Most problems do not explicitly list a par value for bonds. Even though a bond can have any par value, in general, corporate bonds in the United States will have a par value of \$1,000. We will use this par value in all problems unless a different par value is explicitly stated.

Basic

1. The price of a pure discount (zero coupon) bond is the present value of the par. Remember, even though there are no coupon payments, the periods are semiannual to stay consistent with coupon bond payments. So, the price of the bond for each YTM is:

a. $P = \$1,000 / (1 + .05/2)^{20} = \610.27

b. $P = \$1,000 / (1 + .10/2)^{20} = \376.89

c. $P = \$1,000 / (1 + .15/2)^{20} = \235.41

2. The price of any bond is the PV of the interest payment, plus the PV of the par value. Notice this problem assumes a semiannual coupon. The price of the bond at each YTM will be:

a. $P = \$35(\{1 - [1/(1 + .035)]^{50}\} / .035) + \$1,000[1 / (1 + .035)^{50}]$
 $P = \$1,000.00$

When the YTM and the coupon rate are equal, the bond will sell at par.

b. $P = \$35(\{1 - [1/(1 + .045)]^{50}\} / .045) + \$1,000[1 / (1 + .045)^{50}]$
 $P = \$802.38$

When the YTM is greater than the coupon rate, the bond will sell at a discount.

c. $P = \$35(\{1 - [1/(1 + .025)]^{50}\} / .025) + \$1,000[1 / (1 + .025)^{50}]$
 $P = \$1,283.62$

When the YTM is less than the coupon rate, the bond will sell at a premium.

We would like to introduce shorthand notation here. Rather than write (or type, as the case may be) the entire equation for the PV of a lump sum, or the PVA equation, it is common to abbreviate the equations as:

$$PVIF_{R,t} = 1 / (1 + r)^t$$

which stands for Present Value Interest Factor

$$PVIFA_{R,t} = (\{1 - [1/(1 + r)]^t\} / r)$$

which stands for Present Value Interest Factor of an Annuity

These abbreviations are short hand notation for the equations in which the interest rate and the number of periods are substituted into the equation and solved. We will use this shorthand notation in the remainder of the solutions key.

3. Here we are finding the YTM of a semiannual coupon bond. The bond price equation is:

$$P = \$1,050 = \$39(PVIFA_{R\%,20}) + \$1,000(PVIF_{R\%,20})$$

Since we cannot solve the equation directly for R , using a spreadsheet, a financial calculator, or trial and error, we find:

$$R = 3.547\%$$

Since the coupon payments are semiannual, this is the semiannual interest rate. The YTM is the APR of the bond, so:

$$YTM = 2 \times 3.547\% = 7.09\%$$

4. Here we need to find the coupon rate of the bond. All we need to do is to set up the bond pricing equation and solve for the coupon payment as follows:

$$P = \$1,175 = C(PVIFA_{3.8\%,27}) + \$1,000(PVIF_{3.8\%,27})$$

Solving for the coupon payment, we get:

$$C = \$48.48$$

Since this is the semiannual payment, the annual coupon payment is:

$$2 \times \$48.48 = \$96.96$$

And the coupon rate is the annual coupon payment divided by par value, so:

$$\text{Coupon rate} = \$96.96 / \$1,000 = .09696 \text{ or } 9.70\%$$

5. The price of any bond is the PV of the interest payment, plus the PV of the par value. The fact that the bond is denominated in euros is irrelevant. Notice this problem assumes an annual coupon. The price of the bond will be:

$$P = \text{€}84(\{1 - [1/(1 + .076)]^{15}\} / .076) + \text{€}1,000[1 / (1 + .076)^{15}]$$

$$P = \text{€}1,070.18$$

6. Here we are finding the YTM of an annual coupon bond. The fact that the bond is denominated in yen is irrelevant. The bond price equation is:

$$P = ¥87,000 = ¥5,400(PVIFA_{R\%,21}) + ¥100,000(PVIF_{R\%,21})$$

Since we cannot solve the equation directly for R , using a spreadsheet, a financial calculator, or trial and error, we find:

$$R = 6.56\%$$

Since the coupon payments are annual, this is the yield to maturity.

7. The approximate relationship between nominal interest rates (R), real interest rates (r), and inflation (h) is:

$$R = r + h$$

$$\text{Approximate } r = .05 - .039 = .011 \text{ or } 1.10\%$$

The Fisher equation, which shows the exact relationship between nominal interest rates, real interest rates, and inflation is:

$$(1 + R) = (1 + r)(1 + h)$$

$$(1 + .05) = (1 + r)(1 + .039)$$

$$\text{Exact } r = [(1 + .05) / (1 + .039)] - 1 = .0106 \text{ or } 1.06\%$$

8. The Fisher equation, which shows the exact relationship between nominal interest rates, real interest rates, and inflation, is:

$$(1 + R) = (1 + r)(1 + h)$$

$$R = (1 + .025)(1 + .047) - 1 = .0732 \text{ or } 7.32\%$$

9. The Fisher equation, which shows the exact relationship between nominal interest rates, real interest rates, and inflation, is:

$$(1 + R) = (1 + r)(1 + h)$$

$$h = [(1 + .17) / (1 + .11)] - 1 = .0541 \text{ or } 5.41\%$$

10. The Fisher equation, which shows the exact relationship between nominal interest rates, real interest rates, and inflation, is:

$$(1 + R) = (1 + r)(1 + h)$$

$$r = [(1 + .141) / (1.068)] - 1 = .0684 \text{ or } 6.84\%$$

11. The coupon rate, located in the first column of the quote is 6.125%. The bid price is:

$$\text{Bid price} = 119:19 = 119 \frac{19}{32} = 119.59375\% \times \$1,000 = \$1,195.9375$$

The previous day's ask price is found by:

$$\text{Previous day's asked price} = \text{Today's asked price} - \text{Change} = 119 \frac{21}{32} - (-17/32) = 120 \frac{6}{32}$$

The previous day's price in dollars was:

$$\text{Previous day's dollar price} = 120.1875\% \times \$1,000 = \$1,201.875$$

12. This is a premium bond because it sells for more than 100% of face value. The current yield is:

$$\text{Current yield} = \text{Annual coupon payment} / \text{Asked price} = \$75 / \$1,347.1875 = .0557 \text{ or } 5.57\%$$

The YTM is located under the "Asked yield" column, so the YTM is 4.4817%.

The bid-ask spread is the difference between the bid price and the ask price, so:

$$\text{Bid-Ask spread} = 134:23 - 134:22 = 1/32$$

Intermediate

13. Here we are finding the YTM of semiannual coupon bonds for various maturity lengths. The bond price equation is:

$$P = C(\text{PVIFA}_{R\%,t}) + \$1,000(\text{PVIF}_{R\%,t})$$

Miller Corporation bond:

$$\begin{aligned} P_0 &= \$45(\text{PVIFA}_{3.5\%,26}) + \$1,000(\text{PVIF}_{3.5\%,26}) = \$1,168.90 \\ P_1 &= \$45(\text{PVIFA}_{3.5\%,24}) + \$1,000(\text{PVIF}_{3.5\%,24}) = \$1,160.58 \\ P_3 &= \$45(\text{PVIFA}_{3.5\%,20}) + \$1,000(\text{PVIF}_{3.5\%,20}) = \$1,142.12 \\ P_8 &= \$45(\text{PVIFA}_{3.5\%,10}) + \$1,000(\text{PVIF}_{3.5\%,10}) = \$1,083.17 \\ P_{12} &= \$45(\text{PVIFA}_{3.5\%,2}) + \$1,000(\text{PVIF}_{3.5\%,2}) = \$1,019.00 \\ P_{13} &= \$1,000 \end{aligned}$$

Modigliani Company bond:

$$\begin{aligned} P_0 &= \$35(\text{PVIFA}_{4.5\%,26}) + \$1,000(\text{PVIF}_{4.5\%,26}) = \$848.53 \\ P_1 &= \$35(\text{PVIFA}_{4.5\%,24}) + \$1,000(\text{PVIF}_{4.5\%,24}) = \$855.05 \\ P_3 &= \$35(\text{PVIFA}_{4.5\%,20}) + \$1,000(\text{PVIF}_{4.5\%,20}) = \$869.92 \\ P_8 &= \$35(\text{PVIFA}_{4.5\%,10}) + \$1,000(\text{PVIF}_{4.5\%,10}) = \$920.87 \\ P_{12} &= \$35(\text{PVIFA}_{4.5\%,2}) + \$1,000(\text{PVIF}_{4.5\%,2}) = \$981.27 \\ P_{13} &= \$1,000 \end{aligned}$$

All else held equal, the premium over par value for a premium bond declines as maturity approaches, and the discount from par value for a discount bond declines as maturity approaches. This is called "pull to par." In both cases, the largest percentage price changes occur at the shortest maturity lengths.

Also, notice that the price of each bond when no time is left to maturity is the par value, even though the purchaser would receive the par value plus the coupon payment immediately. This is because we calculate the clean price of the bond.

14. Any bond that sells at par has a YTM equal to the coupon rate. Both bonds sell at par, so the initial YTM on both bonds is the coupon rate, 8 percent. If the YTM suddenly rises to 10 percent:

$$P_{\text{Laurel}} = \$40(\text{PVIFA}_{5\%,4}) + \$1,000(\text{PVIF}_{5\%,4}) = \$964.54$$

$$P_{\text{Hardy}} = \$40(\text{PVIFA}_{5\%,30}) + \$1,000(\text{PVIF}_{5\%,30}) = \$846.28$$

The percentage change in price is calculated as:

$$\text{Percentage change in price} = (\text{New price} - \text{Original price}) / \text{Original price}$$

$$\Delta P_{\text{Laurel}}\% = (\$964.54 - 1,000) / \$1,000 = -0.0355 \text{ or } -3.55\%$$

$$\Delta P_{\text{Hardy}}\% = (\$846.28 - 1,000) / \$1,000 = -0.1537 \text{ or } -15.37\%$$

If the YTM suddenly falls to 6 percent:

$$P_{\text{Laurel}} = \$40(\text{PVIFA}_{3\%,4}) + \$1,000(\text{PVIF}_{3\%,4}) = \$1,037.17$$

$$P_{\text{Hardy}} = \$40(\text{PVIFA}_{3\%,30}) + \$1,000(\text{PVIF}_{3\%,30}) = \$1,196.00$$

$$\Delta P_{\text{Laurel}}\% = (\$1,037.17 - 1,000) / \$1,000 = +0.0372 \text{ or } 3.72\%$$

$$\Delta P_{\text{Hardy}}\% = (\$1,196.00 - 1,000) / \$1,000 = +0.1960 \text{ or } 19.60\%$$

All else the same, the longer the maturity of a bond, the greater is its price sensitivity to changes in interest rates. Notice also that for the same interest rate change, the gain from a decline in interest rates is larger than the loss from the same magnitude change. For a plain vanilla bond, this is always true.

15. Initially, at a YTM of 10 percent, the prices of the two bonds are:

$$P_{\text{Faulk}} = \$30(\text{PVIFA}_{5\%,16}) + \$1,000(\text{PVIF}_{5\%,16}) = \$783.24$$

$$P_{\text{Gonas}} = \$70(\text{PVIFA}_{5\%,16}) + \$1,000(\text{PVIF}_{5\%,16}) = \$1,216.76$$

If the YTM rises from 10 percent to 12 percent:

$$P_{\text{Faulk}} = \$30(\text{PVIFA}_{6\%,16}) + \$1,000(\text{PVIF}_{6\%,16}) = \$696.82$$

$$P_{\text{Gonas}} = \$70(\text{PVIFA}_{6\%,16}) + \$1,000(\text{PVIF}_{6\%,16}) = \$1,101.06$$

The percentage change in price is calculated as:

Percentage change in price = (New price – Original price) / Original price

$$\Delta P_{\text{Faulk}} \% = (\$696.82 - 783.24) / \$783.24 = -0.1103 \text{ or } -11.03\%$$

$$\Delta P_{\text{Gonas}} \% = (\$1,101.06 - 1,216.76) / \$1,216.76 = -0.0951 \text{ or } -9.51\%$$

If the YTM declines from 10 percent to 8 percent:

$$P_{\text{Faulk}} = \$30(\text{PVIFA}_{4\%,16}) + \$1,000(\text{PVIF}_{4\%,16}) = \$883.48$$

$$P_{\text{Gonas}} = \$70(\text{PVIFA}_{4\%,16}) + \$1,000(\text{PVIF}_{4\%,16}) = \$1,349.57$$

$$\Delta P_{\text{Faulk}} \% = (\$883.48 - 783.24) / \$783.24 = +0.1280 \text{ or } 12.80\%$$

$$\Delta P_{\text{Gonas}} \% = (\$1,349.57 - 1,216.76) / \$1,216.76 = +0.1092 \text{ or } 10.92\%$$

All else the same, the lower the coupon rate on a bond, the greater is its price sensitivity to changes in interest rates.

16. The bond price equation for this bond is:

$$P_0 = \$960 = \$37(\text{PVIFA}_{R\%,18}) + \$1,000(\text{PVIF}_{R\%,18})$$

Using a spreadsheet, financial calculator, or trial and error we find:

$$R = 4.016\%$$

This is the semiannual interest rate, so the YTM is:

$$\text{YTM} = 2 \times 4.016\% = 8.03\%$$

The current yield is:

$$\text{Current yield} = \text{Annual coupon payment} / \text{Price} = \$74 / \$960 = .0771 \text{ or } 7.71\%$$

The effective annual yield is the same as the EAR, so using the EAR equation from the previous chapter:

$$\text{Effective annual yield} = (1 + 0.04016)^2 - 1 = .0819 \text{ or } 8.19\%$$

17. The company should set the coupon rate on its new bonds equal to the required return. The required return can be observed in the market by finding the YTM on outstanding bonds of the company. So, the YTM on the bonds currently sold in the market is:

$$P = \$1,063 = \$50(\text{PVIFA}_{R\%,40}) + \$1,000(\text{PVIF}_{R\%,40})$$

Using a spreadsheet, financial calculator, or trial and error we find:

$$R = 4.650\%$$

This is the semiannual interest rate, so the YTM is:

$$\text{YTM} = 2 \times 4.650\% = 9.30\%$$

18. Accrued interest is the coupon payment for the period times the fraction of the period that has passed since the last coupon payment. Since we have a semiannual coupon bond, the coupon payment per six months is one-half of the annual coupon payment. There are two months until the next coupon payment, so four months have passed since the last coupon payment. The accrued interest for the bond is:

$$\text{Accrued interest} = \$84/2 \times 4/6 = \$28$$

And we calculate the clean price as:

$$\text{Clean price} = \text{Dirty price} - \text{Accrued interest} = \$1,090 - 28 = \$1,062$$

19. Accrued interest is the coupon payment for the period times the fraction of the period that has passed since the last coupon payment. Since we have a semiannual coupon bond, the coupon payment per six months is one-half of the annual coupon payment. There are four months until the next coupon payment, so two months have passed since the last coupon payment. The accrued interest for the bond is:

$$\text{Accrued interest} = \$72/2 \times 2/6 = \$12.00$$

And we calculate the dirty price as:

$$\text{Dirty price} = \text{Clean price} + \text{Accrued interest} = \$904 + 12 = \$916.00$$

20. To find the number of years to maturity for the bond, we need to find the price of the bond. Since we already have the coupon rate, we can use the bond price equation, and solve for the number of years to maturity. We are given the current yield of the bond, so we can calculate the price as:

$$\text{Current yield} = .0842 = \$90/P_0$$

$$P_0 = \$90/.0842 = \$1,068.88$$

Now that we have the price of the bond, the bond price equation is:

$$P = \$1,068.88 = \$90\{[(1 - (1/1.0781)^t] / .0781\} + \$1,000/1.0781^t$$

We can solve this equation for t as follows:

$$\$1,068.88 (1.0781)^t = \$1,152.37 (1.0781)^t - 1,152.37 + 1,000$$

$$152.37 = 83.49(1.0781)^t$$

$$1.8251 = 1.0781^t$$

$$t = \log 1.8251 / \log 1.0781 = 8.0004 \approx 8 \text{ years}$$

The bond has 8 years to maturity.

21. The bond has 10 years to maturity, so the bond price equation is:

$$P = \$871.55 = \$41.25(PVIFA_{R\%,20}) + \$1,000(PVIF_{R\%,20})$$

Using a spreadsheet, financial calculator, or trial and error we find:

$$R = 5.171\%$$

This is the semiannual interest rate, so the YTM is:

$$YTM = 2 \times 5.171\% = 10.34\%$$

The current yield is the annual coupon payment divided by the bond price, so:

$$\text{Current yield} = \$82.50 / \$871.55 = .0947 \text{ or } 9.47\%$$

22. We found the maturity of a bond in Problem 20. However, in this case, the maturity is indeterminate. A bond selling at par can have any length of maturity. In other words, when we solve the bond pricing equation as we did in Problem 20, the number of periods can be any positive number.

Challenge

23. To find the capital gains yield and the current yield, we need to find the price of the bond. The current price of Bond P and the price of Bond P in one year is:

$$P: \quad P_0 = \$90(PVIFA_{7\%,5}) + \$1,000(PVIF_{7\%,5}) = \$1,082.00$$

$$P_1 = \$90(PVIFA_{7\%,4}) + \$1,000(PVIF_{7\%,4}) = \$1,067.74$$

$$\text{Current yield} = \$90 / \$1,082.00 = .0832 \text{ or } 8.32\%$$

The capital gains yield is:

$$\text{Capital gains yield} = (\text{New price} - \text{Original price}) / \text{Original price}$$

$$\text{Capital gains yield} = (\$1,067.74 - \$1,082.00) / \$1,082.00 = -0.0132 \text{ or } -1.32\%$$

The current price of Bond D and the price of Bond D in one year is:

$$D: \quad P_0 = \$50(PVIFA_{7\%,5}) + \$1,000(PVIF_{7\%,5}) = \$918.00$$

$$P_1 = \$50(PVIFA_{7\%,4}) + \$1,000(PVIF_{7\%,4}) = \$932.26$$

$$\text{Current yield} = \$50 / \$918.00 = 0.0545 \text{ or } 5.45\%$$

$$\text{Capital gains yield} = (\$932.26 - \$918.00) / \$918.00 = 0.0155 \text{ or } 1.55\%$$

All else held constant, premium bonds pay a high current income while having price depreciation as maturity nears; discount bonds pay a lower current income but have price appreciation as maturity nears. For either bond, the total return is still 7%, but this return is distributed differently between current income and capital gains.

24. a. The rate of return you expect to earn if you purchase a bond and hold it until maturity is the YTM. The bond price equation for this bond is:

$$P_0 = \$1,140 = \$90(\text{PVIFA}_{R\%,10}) + \$1,000(\text{PVIF}_{R\%,10})$$

Using a spreadsheet, financial calculator, or trial and error we find:

$$R = \text{YTM} = 7.01\%$$

- b. To find our HPY, we need to find the price of the bond in two years. The price of the bond in two years, at the new interest rate, will be:

$$P_2 = \$90(\text{PVIFA}_{6.01\%,8}) + \$1,000(\text{PVIF}_{6.01\%,8}) = \$1,185.87$$

To calculate the HPY, we need to find the interest rate that equates the price we paid for the bond with the cash flows we received. The cash flows we received were \$90 each year for two years, and the price of the bond when we sold it. The equation to find our HPY is:

$$P_0 = \$1,140 = \$90(\text{PVIFA}_{R\%,2}) + \$1,185.87(\text{PVIF}_{R\%,2})$$

Solving for R , we get:

$$R = \text{HPY} = 9.81\%$$

The realized HPY is greater than the expected YTM when the bond was bought because interest rates dropped by 1 percent; bond prices rise when yields fall.

25. The price of any bond (or financial instrument) is the PV of the future cash flows. Even though Bond M makes different coupons payments, to find the price of the bond, we just find the PV of the cash flows. The PV of the cash flows for Bond M is:

$$P_M = \$800(\text{PVIFA}_{4\%,16})(\text{PVIF}_{4\%,12}) + \$1,000(\text{PVIFA}_{4\%,12})(\text{PVIF}_{4\%,28}) + \$20,000(\text{PVIF}_{4\%,40})$$

$$P_M = \$13,117.88$$

Notice that for the coupon payments of \$800, we found the PVA for the coupon payments, and then discounted the lump sum back to today.

Bond N is a zero coupon bond with a \$20,000 par value; therefore, the price of the bond is the PV of the par, or:

$$P_N = \$20,000(\text{PVIF}_{4\%,40}) = \$4,165.78$$

26. To find the present value, we need to find the real weekly interest rate. To find the real return, we need to use the effective annual rates in the Fisher equation. So, we find the real EAR is:

$$(1 + R) = (1 + r)(1 + h)$$

$$1 + .107 = (1 + r)(1 + .035)$$

$$r = .0696 \text{ or } 6.96\%$$

Now, to find the weekly interest rate, we need to find the APR. Using the equation for discrete compounding:

$$\text{EAR} = [1 + (\text{APR} / m)]^m - 1$$

We can solve for the APR. Doing so, we get:

$$\begin{aligned}\text{APR} &= m[(1 + \text{EAR})^{1/m} - 1] \\ \text{APR} &= 52[(1 + .0696)^{1/52} - 1] \\ \text{APR} &= .0673 \text{ or } 6.73\%\end{aligned}$$

So, the weekly interest rate is:

$$\begin{aligned}\text{Weekly rate} &= \text{APR} / 52 \\ \text{Weekly rate} &= .0673 / 52 \\ \text{Weekly rate} &= .0013 \text{ or } 0.13\%\end{aligned}$$

Now we can find the present value of the cost of the roses. The real cash flows are an ordinary annuity, discounted at the real interest rate. So, the present value of the cost of the roses is:

$$\begin{aligned}\text{PVA} &= C(\{1 - [1/(1 + r)]^t\} / r) \\ \text{PVA} &= \$8(\{1 - [1/(1 + .0013)]^{30(52)}\} / .0013) \\ \text{PVA} &= \$5,359.64\end{aligned}$$

27. To answer this question, we need to find the monthly interest rate, which is the APR divided by 12. We also must be careful to use the real interest rate. The Fisher equation uses the effective annual rate, so, the real effective annual interest rates, and the monthly interest rates for each account are:

Stock account:

$$\begin{aligned}(1 + R) &= (1 + r)(1 + h) \\ 1 + .12 &= (1 + r)(1 + .04) \\ r &= .0769 \text{ or } 7.69\%\end{aligned}$$

$$\begin{aligned}\text{APR} &= m[(1 + \text{EAR})^{1/m} - 1] \\ \text{APR} &= 12[(1 + .0769)^{1/12} - 1] \\ \text{APR} &= .0743 \text{ or } 7.43\%\end{aligned}$$

$$\begin{aligned}\text{Monthly rate} &= \text{APR} / 12 \\ \text{Monthly rate} &= .0743 / 12 \\ \text{Monthly rate} &= .0062 \text{ or } 0.62\%\end{aligned}$$

Bond account:

$$\begin{aligned}(1 + R) &= (1 + r)(1 + h) \\ 1 + .07 &= (1 + r)(1 + .04) \\ r &= .0288 \text{ or } 2.88\%\end{aligned}$$

$$\begin{aligned}\text{APR} &= m[(1 + \text{EAR})^{1/m} - 1] \\ \text{APR} &= 12[(1 + .0288)^{1/12} - 1] \\ \text{APR} &= .0285 \text{ or } 2.85\%\end{aligned}$$

Monthly rate = APR / 12
 Monthly rate = .0285 / 12
 Monthly rate = .0024 or 0.24%

Now we can find the future value of the retirement account in real terms. The future value of each account will be:

Stock account:

$$\begin{aligned} FVA &= C \{ (1 + r)^t - 1 \} / r \\ FVA &= \$800 \{ [(1 + .0062)^{360} - 1] / .0062 \} \\ FVA &= \$1,063,761.75 \end{aligned}$$

Bond account:

$$\begin{aligned} FVA &= C \{ (1 + r)^t - 1 \} / r \\ FVA &= \$400 \{ [(1 + .0024)^{360} - 1] / .0024 \} \\ FVA &= \$227,089.04 \end{aligned}$$

The total future value of the retirement account will be the sum of the two accounts, or:

$$\begin{aligned} \text{Account value} &= \$1,063,761.75 + 227,089.04 \\ \text{Account value} &= \$1,290,850.79 \end{aligned}$$

Now we need to find the monthly interest rate in retirement. We can use the same procedure that we used to find the monthly interest rates for the stock and bond accounts, so:

$$\begin{aligned} (1 + R) &= (1 + r)(1 + h) \\ 1 + .08 &= (1 + r)(1 + .04) \\ r &= .0385 \text{ or } 3.85\% \end{aligned}$$

$$\begin{aligned} \text{APR} &= m[(1 + \text{EAR})^{1/m} - 1] \\ \text{APR} &= 12[(1 + .0385)^{1/12} - 1] \\ \text{APR} &= .0378 \text{ or } 3.78\% \end{aligned}$$

Monthly rate = APR / 12
 Monthly rate = .0378 / 12
 Monthly rate = .0031 or 0.31%

Now we can find the real monthly withdrawal in retirement. Using the present value of an annuity equation and solving for the payment, we find:

$$\begin{aligned} PVA &= C(\{1 - [1/(1 + r)]^t\} / r) \\ \$1,290,850.79 &= C(\{1 - [1/(1 + .0031)]^{300}\} / .0031) \\ C &= \$6,657.74 \end{aligned}$$

This is the real dollar amount of the monthly withdrawals. The nominal monthly withdrawals will increase by the inflation rate each month. To find the nominal dollar amount of the last withdrawal, we can increase the real dollar withdrawal by the inflation rate. We can increase the real withdrawal by the effective annual inflation rate since we are only interested in the nominal amount of the last withdrawal. So, the last withdrawal in nominal terms will be:

$$\begin{aligned} FV &= PV(1 + r)^t \\ FV &= \$6,657.74(1 + .04)^{(30 + 25)} \\ FV &= \$57,565.30 \end{aligned}$$

28. In this problem, we need to calculate the future value of the annual savings after the five years of operations. The savings are the revenues minus the costs, or:

$$\text{Savings} = \text{Revenue} - \text{Costs}$$

Since the annual fee and the number of members are increasing, we need to calculate the effective growth rate for revenues, which is:

$$\begin{aligned} \text{Effective growth rate} &= (1 + .06)(1 + .03) - 1 \\ \text{Effective growth rate} &= .0918 \text{ or } 9.18\% \end{aligned}$$

The revenue for the current year is the number of members times the annual fee, or:

$$\begin{aligned} \text{Current revenue} &= 500(\$500) \\ \text{Current revenue} &= \$250,000 \end{aligned}$$

The revenue will grow at 9.18 percent, and the costs will grow at 2 percent, so the savings each year for the next five years will be:

<u>Year</u>	<u>Revenue</u>	<u>Costs</u>	<u>Savings</u>
1	\$ 272,950.00	\$ 76,500.00	\$ 196,450.00
2	298,006.81	78,030.00	219,976.81
3	325,363.84	79,590.60	245,773.24
4	355,232.24	81,182.41	274,049.82
5	387,842.55	82,806.06	305,036.49

Now we can find the value of each year's savings using the future value of a lump sum equation, so:

$$FV = PV(1 + r)^t$$

<u>Year</u>		<u>Future Value</u>
1	$\$196,450.00(1 + .09)^4 =$	\$277,305.21
2	$\$219,976.81(1 + .09)^3 =$	284,876.35
3	$\$245,773.24(1 + .09)^2 =$	292,003.18
4	$\$274,049.82(1 + .09)^1 =$	298,714.31
5		<u>305,036.49</u>
Total future value of savings =		\$1,457,935.54

He will spend \$500,000 on a luxury boat, so the value of his account will be:

Value of account = $\$1,457,935.54 - 500,000$

Value of account = \$957,935.54

Now we can use the present value of an annuity equation to find the payment. Doing so, we find:

$$PVA = C(\{1 - [1/(1 + r)]^t\} / r)$$

$$\$957,935.54 = C(\{1 - [1/(1 + .09)]^{25}\} / .09)$$

$$C = \$97,523.83$$

Calculator Solutions

1.

a.

Enter	20	2.5%			\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$610.27		

b.

Enter	20	5%			\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$376.89		

c.

Enter	20	7.5%			\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$235.41		

2.

a.

Enter	50	3.5%		\$35	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$1,000.00		

b.

Enter	50	4.5%		\$35	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$802.38		

c.

Enter	50	2.5%		\$35	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$1,283.62		

3.

Enter	20		±\$1,050	\$39	\$1,000
	N	I/Y	PV	PMT	FV
Solve for		3.547%			

$3.547\% \times 2 = 7.09\%$

4.

Enter	27	3.8%	±\$1,175		\$1,000
	N	I/Y	PV	PMT	FV
Solve for				\$48.48	

$\$48.48 \times 2 = \96.96
 $\$96.96 / \$1,000 = 9.70\%$

5.

Enter	15	7.60%		€84	€1,000
	N	I/Y	PV	PMT	FV
Solve for			€1,070.18		

6.

Enter	21		±¥87,000	¥5,400	¥100,000
	N	I/Y	PV	PMT	FV
Solve for		6.56%			

13. Miller Corporation

P_0					
Enter	26	3.5%		\$45	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$1,168.90		

P_1					
Enter	24	3.5%		\$45	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$1,160.58		

P_3					
Enter	20	3.5%		\$45	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$1,142.12		

P_8					
Enter	10	3.5%		\$45	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$1,083.17		

P_{12}					
Enter	2	3.5%		\$45	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$1,019.00		

Modigliani Company

P_0					
Enter	26	4.5%		\$35	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$848.53		

P_1					
Enter	24	4.5%		\$35	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$855.05		

Solve for	\$981.72
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224

P_{Faulk}					
Enter	16	5%		\$30	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$783.24		

P_{Gonas}					
Enter	16	5%		\$70	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$1,216.76		

P _{Fault}					
Enter	16	6%		\$30	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$696.82		
$\Delta P_{\text{Fault}}\% = (\$696.82 - 783.24) / \$783.24 = -11.03\%$					

P _{Gonas}	16	6%		\$70	\$1,000
Enter	N	I/Y	PV	PMT	FV
Solve for			\$1,101.06		
$\Delta P_{Gonas} \% = (\$1,101.06 - \$1,216.76) / \$1,216.76 = -9.51\%$					

P _{Fault}	16	4%		\$30	\$1,000
Enter	N	I/Y	PV	PMT	FV
Solve for			\$883.48		
$\Delta P_{\text{Fault}}\% = (\$883.48 - 783.24) / \$783.24 = +12.80\%$					

P _{Gonas}					
Enter	16	4%		\$70	\$1,000
	N	I/Y	PV	PMT	FV
Solve for			\$1,349.57		
$\Delta P_{Gonas} \% = (\$1,349.57 - \$1,216.76) / \$1,216.76 = +10.92\%$					

16.

Enter	18		±\$960	\$37	\$1,000
	N	I/Y	PV	PMT	FV
Solve for		4.016%			
	YTM = 4.016% × 2 = 8.03%				

17. The company should set the coupon rate on its new bonds equal to the required return; the required return can be observed in the market by finding the YTM on outstanding bonds of the company.

Enter 40 ±\$1,063 \$50 \$1,000
 N **I/Y** **PV** **PMT** **FV**
 Solve for 4.650%
 $4.650\% \times 2 = 9.30\%$

20. Current yield = $.0842 = \$90/P_0$; $P_0 = \$1,068.88$

Enter 7.81% ±\$1,068.88 \$90 \$1,000
 N **I/Y** **PV** **PMT** **FV**
 Solve for 8.0004
 8 years

- 21.

Enter 20 ±\$871.55 \$41.25 \$1,000
 N **I/Y** **PV** **PMT** **FV**
 Solve for 5.171%
 $5.171\% \times 2 = 10.34\%$

- 23.

Bond P

P_0

Enter 5 7% \$90 \$1,000
 N **I/Y** **PV** **PMT** **FV**
 Solve for \$1,082.00

P_1

Enter 4 7% \$90 \$1,000
 N **I/Y** **PV** **PMT** **FV**
 Solve for \$1,067.74

$$\text{Current yield} = \$90 / \$1,082.00 = 8.32\%$$

$$\text{Capital gains yield} = (\$1,067.74 - \$1,082.00) / \$1,082.00 = -1.32\%$$

Bond D

P_0

Enter 5 7% \$50 \$1,000
 N **I/Y** **PV** **PMT** **FV**
 Solve for \$918.00

P_1

Enter 4 7% \$50 \$1,000
 N **I/Y** **PV** **PMT** **FV**
 Solve for \$932.26

$$\text{Current yield} = \$50 / \$918.00 = 5.45\%$$

$$\text{Capital gains yield} = (\$932.26 - \$918.00) / \$918.00 = +1.55\%$$

All else held constant, premium bonds pay a higher current income while having price depreciation as maturity nears; discount bonds pay a lower current income but have price appreciation as maturity nears. For either bond, the total return is still 7%, but this return is distributed differently between current income and capital gains.

24.

a.

Enter	10		±\$1,140	\$90	\$1,000
	N	I/Y	PV	PMT	FV

Solve for 7.01%

This is the rate of return you expect to earn on your investment when you purchase the bond.

b.

Enter	8	6.01%		\$90	\$1,000
	N	I/Y	PV	PMT	FV

Solve for \$1,185.87

The HPY is:

Enter	2		±\$1,140	\$90	\$1,185.17
	N	I/Y	PV	PMT	FV

Solve for 9.81%

The realized HPY is greater than the expected YTM when the bond was bought because interest rates dropped by 1 percent; bond prices rise when yields fall.

25.

P_M

CFo	\$0
C01	\$0
F01	12
C02	\$800
F02	16
C03	\$1,000
F03	11
C04	\$21,000
F04	1

I = 4%

NPV CPT

\$13,117.88

P_N

Enter	40	4%			\$20,000
	N	I/Y	PV	PMT	FV

Solve for \$4,165.78

29.

Real return for stock account: $1 + .12 = (1 + r)(1 + .04)$; $r = 7.6923\%$

Enter	7.6923%	12
	NOM	C/Y

Solve for 7.4337%

Real return for bond account: $1 + .07 = (1 + r)(1 + .04)$; $r = 2.8846\%$

Enter	2.8846%	12
	NOM	C/Y

Solve for 2.8472%

Enter	3.8462%	12
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NOM **EFF** **C/Y**

Solve for **3.7800%**

Enter 12×30 7.4337% / 12

Solve for	N	I/Y	PV	PMT	FV
					\$1,063,761.75

Solve for _____ \$1,063,761.75

Enter	12×30	$2.8472\% / 12$
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Solve for	N	I/Y	PV	PMT	FV
					\$227,089.04

Solve for _____ \$227,089.04

$$\text{Retirement value} = \$1,063,761.75 + 227,089.04 = \$1,290,850.79$$

Enter	25×12	$3.7800\% / 12$	\$1,290,850.79
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	N	I/Y	PV	PMT	FV
Solve for				\$6,657.74	

Solve for $\frac{A}{B}$, $\frac{C}{D}$, $\frac{E}{F}$, $\frac{G}{H}$

\$6,657.74

Enter	30 + 25	4%	\$6,657.74
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Solve for	N	I/Y	PV	PMT	FV
					\$57,565.30

Solve for PV FV PMT Rate Nper

\$57,565.30

Future value of savings:

Enter	4	9%	\$196,450
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Solve for	N	I/Y	PV	PMT	FV
					\$277,305.21

Solve for _____ \$277,305.21

Enter	3	9%	\$219,976.81
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	N	I/Y	PV	PMT	FV
Solve for					\$284,876.35

Solve for \$284,876.35

Enter	2	9%	\$245,773.24
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Solve for	N	I/Y	PV	PMT	FV
					\$292,003.18

Solve for x y z w \$292,003.18

Enter	1	9%	\$274,049.82
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Solve for	N	I/Y	PV	PMT	FV
					\$298,714.31

Solve for _____ \$298,714.31

$\text{Future value} = \$277,305.21 + 284,876.35 + 292,003.18 + 298,714.31 + 305,036.49$
 $\text{Future value} = \$1,457,935.54$

He will spend \$500,000 on a luxury boat, so the value of his account will be:

$\text{Value of account} = \$1,457,935.54 - 500,000$
 $\text{Value of account} = \$957,935.54$

Enter	25	9%	\$957,935.54		
	N	I/Y	PV	PMT	FV
Solve for				\$97,523.83	

CHAPTER 9

HOW TO VALUE STOCKS

Answers to Concept Questions

1. The value of any investment depends on the present value of its cash flows; i.e., what investors will actually receive. The cash flows from a share of stock are the dividends.
2. Investors believe the company will eventually start paying dividends (or be sold to another company).
3. In general, companies that need the cash will often forgo dividends since dividends are a cash expense. Young, growing companies with profitable investment opportunities are one example; another example is a company in financial distress. This question is examined in depth in a later chapter.
4. The general method for valuing a share of stock is to find the present value of all expected future dividends. The dividend growth model presented in the text is only valid (i) if dividends are expected to occur forever; that is, the stock provides dividends in perpetuity, and (ii) if a constant growth rate of dividends occurs forever. A violation of the first assumption might be a company that is expected to cease operations and dissolve itself some finite number of years from now. The stock of such a company would be valued by applying the general method of valuation explained in this chapter. A violation of the second assumption might be a start-up firm that isn't currently paying any dividends, but is expected to eventually start making dividend payments some number of years from now. This stock would also be valued by the general dividend valuation method explained in this chapter.
5. The common stock probably has a higher price because the dividend can grow, whereas it is fixed on the preferred. However, the preferred is less risky because of the dividend and liquidation preference, so it is possible the preferred could be worth more, depending on the circumstances.
6. The two components are the dividend yield and the capital gains yield. For most companies, the capital gains yield is larger. This is easy to see for companies that pay no dividends. For companies that do pay dividends, the dividend yields are rarely over five percent and are often much less.
7. Yes. If the dividend grows at a steady rate, so does the stock price. In other words, the dividend growth rate and the capital gains yield are the same.
8. The three factors are: 1) The company's future growth opportunities. 2) The company's level of risk, which determines the interest rate used to discount cash flows. 3) The accounting method used.
9. It wouldn't seem to be. Investors who don't like the voting features of a particular class of stock are under no obligation to buy it.
10. Presumably, the current stock value reflects the risk, timing and magnitude of all future cash flows, both short-term and long-term. If this is correct, then the statement is false.

Solutions to Questions and Problems

NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

Basic

1. The constant dividend growth model is:

$$P_t = D_t \times (1 + g) / (R - g)$$

So, the price of the stock today is:

$$P_0 = D_0 (1 + g) / (R - g) = \$1.90 (1.05) / (.12 - .05) = \$28.50$$

The dividend at year 4 is the dividend today times the FVIF for the growth rate in dividends and four years, so:

$$P_3 = D_3 (1 + g) / (R - g) = D_0 (1 + g)^4 / (R - g) = \$1.90 (1.05)^4 / (.12 - .05) = \$32.99$$

We can do the same thing to find the dividend in Year 16, which gives us the price in Year 15, so:

$$P_{15} = D_{15} (1 + g) / (R - g) = D_0 (1 + g)^{16} / (R - g) = \$1.90 (1.05)^{16} / (.12 - .05) = \$59.25$$

There is another feature of the constant dividend growth model: The stock price grows at the dividend growth rate. So, if we know the stock price today, we can find the future value for any time in the future we want to calculate the stock price. In this problem, we want to know the stock price in three years, and we have already calculated the stock price today. The stock price in three years will be:

$$P_3 = P_0(1 + g)^3 = \$28.50(1 + .05)^3 = \$32.99$$

And the stock price in 15 years will be:

$$P_{15} = P_0(1 + g)^{15} = \$28.50(1 + .05)^{15} = \$59.25$$

2. We need to find the required return of the stock. Using the constant growth model, we can solve the equation for R . Doing so, we find:

$$R = (D_1 / P_0) + g = (\$2.85 / \$58) + .06 = .1091 \text{ or } 10.91\%$$

3. The dividend yield is the dividend next year divided by the current price, so the dividend yield is:

$$\text{Dividend yield} = D_1 / P_0 = \$2.85 / \$58 = .0491 \text{ or } 4.91\%$$

The capital gains yield, or percentage increase in the stock price, is the same as the dividend growth rate, so:

$$\text{Capital gains yield} = 6\%$$

4. Using the constant growth model, we find the price of the stock today is:

$$P_0 = D_1 / (R - g) = \$3.05 / (.11 - .0525) = \$53.04$$

5. The required return of a stock is made up of two parts: The dividend yield and the capital gains yield. So, the required return of this stock is:

$$R = \text{Dividend yield} + \text{Capital gains yield} = .047 + .058 = .1050 \text{ or } 10.50\%$$

6. We know the stock has a required return of 13 percent, and the dividend and capital gains yield are equal, so:

$$\text{Dividend yield} = 1/2(.13) = .065 = \text{Capital gains yield}$$

Now we know both the dividend yield and capital gains yield. The dividend is simply the stock price times the dividend yield, so:

$$D_1 = .065(\$64) = \$4.16$$

This is the dividend next year. The question asks for the dividend this year. Using the relationship between the dividend this year and the dividend next year:

$$D_1 = D_0(1 + g)$$

We can solve for the dividend that was just paid:

$$\$4.16 = D_0 (1 + .065)$$

$$D_0 = \$4.16 / 1.065 = \$3.91$$

7. The price of any financial instrument is the PV of the future cash flows. The future dividends of this stock are an annuity for 9 years, so the price of the stock is the PVA, which will be:

$$P_0 = \$11(\text{PVIFA}_{10\%,9}) = \$63.35$$

8. The price of a share of preferred stock is the dividend divided by the required return. This is the same equation as the constant growth model, with a dividend growth rate of zero percent. Remember that most preferred stock pays a fixed dividend, so the growth rate is zero. Using this equation, we find the price per share of the preferred stock is:

$$R = D/P_0 = \$6.40/\$103 = .0621 \text{ or } 6.21\%$$

9. The growth rate of earnings is the return on equity times the retention ratio, so:

$$\begin{aligned} g &= \text{ROE} \times b \\ g &= .15(.70) \\ g &= .1050 \text{ or } 10.50\% \end{aligned}$$

To find next year's earnings, we simply multiply the current earnings times one plus the growth rate, so:

$$\begin{aligned} \text{Next year's earnings} &= \text{Current earnings}(1 + g) \\ \text{Next year's earnings} &= \$28,000,000(1 + .1050) \\ \text{Next year's earnings} &= \$30,940,000 \end{aligned}$$

Intermediate

10. This stock has a constant growth rate of dividends, but the required return changes twice. To find the value of the stock today, we will begin by finding the price of the stock at Year 6, when both the dividend growth rate and the required return are stable forever. The price of the stock in Year 6 will be the dividend in Year 7, divided by the required return minus the growth rate in dividends. So:

$$P_6 = D_6 (1 + g) / (R - g) = D_0 (1 + g)^7 / (R - g) = \$2.75(1.06)^7 / (.11 - .06) = \$82.70$$

Now we can find the price of the stock in Year 3. We need to find the price here since the required return changes at that time. The price of the stock in Year 3 is the PV of the dividends in Years 4, 5, and 6, plus the PV of the stock price in Year 6. The price of the stock in Year 3 is:

$$\begin{aligned} P_3 &= \$2.75(1.06)^4 / 1.14 + \$2.75(1.06)^5 / 1.14^2 + \$2.75(1.06)^6 / 1.14^3 + \$82.70 / 1.14^3 \\ P_3 &= \$64.33 \end{aligned}$$

Finally, we can find the price of the stock today. The price today will be the PV of the dividends in Years 1, 2, and 3, plus the PV of the stock in Year 3. The price of the stock today is:

$$\begin{aligned} P_0 &= \$2.75(1.06) / 1.16 + \$2.75(1.06)^2 / (1.16)^2 + \$2.75(1.06)^3 / (1.16)^3 + \$64.33 / (1.16)^3 \\ P_0 &= \$48.12 \end{aligned}$$

11. Here we have a stock that pays no dividends for 10 years. Once the stock begins paying dividends, it will have a constant growth rate of dividends. We can use the constant growth model at that point. It is important to remember that general form of the constant dividend growth formula is:

$$P_t = [D_t \times (1 + g)] / (R - g)$$

This means that since we will use the dividend in Year 10, we will be finding the stock price in Year 9. The dividend growth model is similar to the PVA and the PV of a perpetuity: The equation gives you the PV one period before the first payment. So, the price of the stock in Year 9 will be:

$$P_9 = D_{10} / (R - g) = \$9.00 / (.13 - .055) = \$120.00$$

The price of the stock today is simply the PV of the stock price in the future. We simply discount the future stock price at the required return. The price of the stock today will be:

$$P_0 = \$120.00 / 1.13^9 = \$39.95$$

12. The price of a stock is the PV of the future dividends. This stock is paying five dividends, so the price of the stock is the PV of these dividends using the required return. The price of the stock is:

$$P_0 = \$13 / 1.11 + \$16 / 1.11^2 + \$19 / 1.11^3 + \$22 / 1.11^4 + \$25 / 1.11^5 = \$67.92$$

13. With differential dividends, we find the price of the stock when the dividends level off at a constant growth rate, and then find the PV of the future stock price, plus the PV of all dividends during the differential growth period. The stock begins constant growth in Year 5, so we can find the price of the stock in Year 4, one year before the constant dividend growth begins, as:

$$P_4 = D_4 (1 + g) / (R - g) = \$2.50(1.05) / (.13 - .05) = \$32.81$$

The price of the stock today is the PV of the first four dividends, plus the PV of the Year 4 stock price. So, the price of the stock today will be:

$$P_0 = \$9 / 1.13 + \$7 / 1.13^2 + \$5 / 1.13^3 + (\$2.50 + 32.81) / 1.13^4 = \$38.57$$

14. With differential dividends, we find the price of the stock when the dividends level off at a constant growth rate, and then find the PV of the future stock price, plus the PV of all dividends during the differential growth period. The stock begins constant growth in Year 4, so we can find the price of the stock in Year 3, one year before the constant dividend growth begins as:

$$P_3 = D_3 (1 + g) / (R - g) = D_0 (1 + g_1)^3 (1 + g_2) / (R - g_2) = \$2.40(1.25)^3(1.07) / (.12 - .07) = \$100.31$$

The price of the stock today is the PV of the first three dividends, plus the PV of the Year 3 stock price. The price of the stock today will be:

$$P_0 = \$2.40(1.25) / 1.12 + \$2.40(1.25)^2 / 1.12^2 + \$2.40(1.25)^3 / 1.12^3 + \$100.31 / 1.12^3$$

$$P_0 = \$80.40$$

15. Here we need to find the dividend next year for a stock experiencing differential growth. We know the stock price, the dividend growth rates, and the required return, but not the dividend. First, we need to realize that the dividend in Year 3 is the current dividend times the FVIF. The dividend in Year 3 will be:

$$D_3 = D_0 (1.30)^3$$

And the dividend in Year 4 will be the dividend in Year 3 times one plus the growth rate, or:

$$D_4 = D_0 (1.30)^3 (1.18)$$

The stock begins constant growth after the 4th dividend is paid, so we can find the price of the stock in Year 4 as the dividend in Year 5, divided by the required return minus the growth rate. The equation for the price of the stock in Year 4 is:

$$P_4 = D_4 (1 + g) / (R - g)$$

Now we can substitute the previous dividend in Year 4 into this equation as follows:

$$P_4 = D_0 (1 + g_1)^3 (1 + g_2) (1 + g_3) / (R - g_3)$$

$$P_4 = D_0 (1.30)^3 (1.18) (1.08) / (.13 - .08) = 56.00D_0$$

When we solve this equation, we find that the stock price in Year 4 is 56.00 times as large as the dividend today. Now we need to find the equation for the stock price today. The stock price today is the PV of the dividends in Years 1, 2, 3, and 4, plus the PV of the Year 4 price. So:

$$P_0 = D_0(1.30)/1.13 + D_0(1.30)^2/1.13^2 + D_0(1.30)^3/1.13^3 + D_0(1.30)^3(1.18)/1.13^4 + 56.00D_0/1.13^4$$

We can factor out D_0 in the equation, and combine the last two terms. Doing so, we get:

$$P_0 = \$65.00 = D_0 \{ 1.30/1.13 + 1.30^2/1.13^2 + 1.30^3/1.13^3 + [(1.30)^3(1.18) + 56.00] / 1.13^4 \}$$

Reducing the equation even further by solving all of the terms in the braces, we get:

$$\$65 = \$39.86D_0$$

$$D_0 = \$65.00 / \$39.86 = \$1.63$$

This is the dividend today, so the projected dividend for the next year will be:

$$D_1 = \$1.63(1.30) = \$2.12$$

16. The constant growth model can be applied even if the dividends are declining by a constant percentage, just make sure to recognize the negative growth. So, the price of the stock today will be:

$$P_0 = D_0 (1 + g) / (R - g) = \$12(1 - .06) / [(.11 - (-.06))] = \$66.35$$

17. We are given the stock price, the dividend growth rate, and the required return, and are asked to find the dividend. Using the constant dividend growth model, we get:

$$P_0 = \$49.80 = D_0 (1 + g) / (R - g)$$

Solving this equation for the dividend gives us:

$$D_0 = \$49.80(.11 - .05) / (1.05) = \$2.85$$

18. The price of a share of preferred stock is the dividend payment divided by the required return. We know the dividend payment in Year 5, so we can find the price of the stock in Year 4, one year before the first dividend payment. Doing so, we get:

$$P_4 = \$7.00 / .06 = \$116.67$$

The price of the stock today is the PV of the stock price in the future, so the price today will be:

$$P_0 = \$116.67 / (1.06)^4 = \$92.41$$

19. The annual dividend is the dividend divided by the stock price, so:

$$\text{Dividend yield} = \text{Dividend} / \text{Stock price}$$

$$.016 = \text{Dividend} / \$19.47$$

$$\text{Dividend} = \$0.31$$

The “Net Chg” of the stock shows the stock decreased by \$0.12 on this day, so the closing stock price yesterday was:

$$\text{Yesterday's closing price} = \$19.47 - (-0.12) = \$19.59$$

To find the net income, we need to find the EPS. The stock quote tells us the P/E ratio for the stock is 19. Since we know the stock price as well, we can use the P/E ratio to solve for EPS as follows:

$$P/E = 19 = \text{Stock price} / \text{EPS} = \$19.47 / \text{EPS}$$

$$\text{EPS} = \$19.47 / 19 = \$1.025$$

We know that EPS is just the total net income divided by the number of shares outstanding, so:

$$\text{EPS} = \text{NI} / \text{Shares} = \$1.025 = \text{NI} / 25,000,000$$

$$\text{NI} = \$1.025(25,000,000) = \$25,618,421$$

20. To find the number of shares owned, we can divide the amount invested by the stock price. The share price of any financial asset is the present value of the cash flows, so, to find the price of the stock we need to find the cash flows. The cash flows are the two dividend payments plus the sale price. We also need to find the aftertax dividends since the assumption is all dividends are taxed at the same rate for all investors. The aftertax dividends are the dividends times one minus the tax rate, so:

$$\text{Year 1 aftertax dividend} = \$1.50(1 - .28)$$

$$\text{Year 1 aftertax dividend} = \$1.08$$

$$\text{Year 2 aftertax dividend} = \$2.25(1 - .28)$$

$$\text{Year 2 aftertax dividend} = \$1.62$$

We can now discount all cash flows from the stock at the required return. Doing so, we find the price of the stock is:

$$P = \$1.08/1.15 + \$1.62/(1.15)^2 + \$60/(1+.15)^3$$

$$P = \$41.62$$

The number of shares owned is the total investment divided by the stock price, which is:

$$\text{Shares owned} = \$100,000 / \$41.62$$

$$\text{Shares owned} = 2,402.98$$

21. Here we have a stock paying a constant dividend for a fixed period, and an increasing dividend thereafter. We need to find the present value of the two different cash flows using the appropriate quarterly interest rate. The constant dividend is an annuity, so the present value of these dividends is:

$$\begin{aligned} PVA &= C(PVIFA_{R,t}) \\ PVA &= \$0.75(PVIFA_{2.5\%,12}) \\ PVA &= \$7.69 \end{aligned}$$

Now we can find the present value of the dividends beyond the constant dividend phase. Using the present value of a growing annuity equation, we find:

$$\begin{aligned} P_{12} &= D_{13} / (R - g) \\ P_{12} &= \$0.75(1 + .01) / (.025 - .01) \\ P_{12} &= \$50.50 \end{aligned}$$

This is the price of the stock immediately after it has paid the last constant dividend. So, the present value of the future price is:

$$\begin{aligned} PV &= \$50.50 / (1 + .025)^{12} \\ PV &= \$37.55 \end{aligned}$$

The price today is the sum of the present value of the two cash flows, so:

$$\begin{aligned} P_0 &= \$7.69 + 37.55 \\ P_0 &= \$45.24 \end{aligned}$$

22. Here we need to find the dividend next year for a stock with nonconstant growth. We know the stock price, the dividend growth rates, and the required return, but not the dividend. First, we need to realize that the dividend in Year 3 is the constant dividend times the FVIF. The dividend in Year 3 will be:

$$D_3 = D(1.05)$$

The equation for the stock price will be the present value of the constant dividends, plus the present value of the future stock price, or:

$$\begin{aligned} P_0 &= D / 1.11 + D / 1.11^2 + D(1.05) / (.11 - .05) / 1.11^2 \\ \$38 &= D / 1.11 + D / 1.11^2 + D(1.05) / (.11 - .05) / 1.11^2 \end{aligned}$$

We can factor out D_0 in the equation. Doing so, we get:

$$\$38 = D \{ 1/1.11 + 1/1.11^2 + [(1.05) / (.11 - .05)] / 1.11^2 \}$$

Reducing the equation even further by solving all of the terms in the braces, we get:

$$\$38 = D(15.9159)$$

$$D = \$38 / 15.9159 = \$2.39$$

23. The required return of a stock consists of two components, the capital gains yield and the dividend yield. In the constant dividend growth model (growing perpetuity equation), the capital gains yield is the same as the dividend growth rate, or algebraically:

$$R = D_1/P_0 + g$$

We can find the dividend growth rate by the growth rate equation, or:

$$g = \text{ROE} \times b$$

$$g = .16 \times .80$$

$$g = .1280 \text{ or } 12.80\%$$

This is also the growth rate in dividends. To find the current dividend, we can use the information provided about the net income, shares outstanding, and payout ratio. The total dividends paid is the net income times the payout ratio. To find the dividend per share, we can divide the total dividends paid by the number of shares outstanding. So:

$$\text{Dividend per share} = (\text{Net income} \times \text{Payout ratio}) / \text{Shares outstanding}$$

$$\text{Dividend per share} = (\$10,000,000 \times .20) / 2,000,000$$

$$\text{Dividend per share} = \$1.00$$

Now we can use the initial equation for the required return. We must remember that the equation uses the dividend in one year, so:

$$R = D_1/P_0 + g$$

$$R = \$1(1 + .1280)/\$85 + .1280$$

$$R = .1413 \text{ or } 14.13\%$$

24. First, we need to find the annual dividend growth rate over the past four years. To do this, we can use the future value of a lump sum equation, and solve for the interest rate. Doing so, we find the dividend growth rate over the past four years was:

$$FV = PV(1 + R)^t$$

$$\$1.93 = \$1.20(1 + R)^4$$

$$R = (\$1.93 / \$1.20)^{1/4} - 1$$

$$R = .1261 \text{ or } 12.61\%$$

We know the dividend will grow at this rate for five years before slowing to a constant rate indefinitely. So, the dividend amount in seven years will be:

$$D_7 = D_0(1 + g_1)^5(1 + g_2)^2$$

$$D_7 = \$1.93(1 + .1261)^5(1 + .07)^2$$

$$D_7 = \$4.00$$

25. a. We can find the price of all the outstanding company stock by using the dividends the same way we would value an individual share. Since earnings are equal to dividends, and there is no growth, the value of the company's stock today is the present value of a perpetuity, so:

$$P = D / R$$

$$P = \$750,000 / .14$$

$$P = \$5,357,142.86$$

The price-earnings ratio is the stock price divided by the current earnings, so the price-earnings ratio of each company with no growth is:

$$P/E = \text{Price} / \text{Earnings}$$

$$P/E = \$5,357,142.86 / \$750,000$$

$$P/E = 7.14 \text{ times}$$

- b. Since the earnings have increased, the price of the stock will increase. The new price of the all the outstanding company stock is:

$$P = D / R$$

$$P = (\$750,000 + 100,000) / .14$$

$$P = \$6,071,428.57$$

The price-earnings ratio is the stock price divided by the current earnings, so the price-earnings with the increased earnings is:

$$P/E = \text{Price} / \text{Earnings}$$

$$P/E = \$6,071,428.57 / \$750,000$$

$$P/E = 8.10 \text{ times}$$

- c. Since the earnings have increased, the price of the stock will increase. The new price of the all the outstanding company stock is:

$$P = D / R$$

$$P = (\$750,000 + 200,000) / .14$$

$$P = \$6,785,714.29$$

The price-earnings ratio is the stock price divided by the current earnings, so the price-earnings with the increased earnings is:

$$P/E = \text{Price} / \text{Earnings}$$

$$P/E = \$6,785,714.29 / \$750,000$$

$$P/E = 9.05 \text{ times}$$

26. a. If the company does not make any new investments, the stock price will be the present value of the constant perpetual dividends. In this case, all earnings are paid dividends, so, applying the perpetuity equation, we get:

$$P = \text{Dividend} / R$$

$$P = \$8.25 / .12$$

$$P = \$68.75$$

- b. The investment is a one-time investment that creates an increase in EPS for two years. To calculate the new stock price, we need the cash cow price plus the NPVGO. In this case, the NPVGO is simply the present value of the investment plus the present value of the increases in EPS. So, the NPVGO will be:

$$\text{NPVGO} = C_1 / (1 + R) + C_2 / (1 + R)^2 + C_3 / (1 + R)^3$$

$$\text{NPVGO} = -\$1.60 / 1.12 + \$2.10 / 1.12^2 + \$2.45 / 1.12^3$$

$$\text{NPVGO} = \$1.99$$

So, the price of the stock if the company undertakes the investment opportunity will be:

$$P = \$68.75 + 1.99$$

$$P = \$70.74$$

- c. After the project is over, and the earnings increase no longer exists, the price of the stock will revert back to \$68.75, the value of the company as a cash cow.
27. a. The price of the stock is the present value of the dividends. Since earnings are equal to dividends, we can find the present value of the earnings to calculate the stock price. Also, since we are excluding taxes, the earnings will be the revenues minus the costs. We simply need to find the present value of all future earnings to find the price of the stock. The present value of the revenues is:

$$PV_{\text{Revenue}} = C_1 / (R - g)$$

$$PV_{\text{Revenue}} = \$6,000,000(1 + .05) / (.15 - .05)$$

$$PV_{\text{Revenue}} = \$63,000,000$$

And the present value of the costs will be:

$$PV_{\text{Costs}} = C_1 / (R - g)$$

$$PV_{\text{Costs}} = \$3,100,000(1 + .05) / (.15 - .05)$$

$$PV_{\text{Costs}} = \$32,550,000$$

Since there are no taxes, the present value of the company's earnings and dividends will be:

$$PV_{\text{Dividends}} = \$63,000,000 - 32,550,000$$

$$PV_{\text{Dividends}} = \$30,450,000$$

Note that since revenues and costs increase at the same rate, we could have found the present value of future dividends as the present value of current dividends. Doing so, we find:

$$D_0 = \text{Revenue}_0 - \text{Costs}_0$$

$$D_0 = \$6,000,000 - 3,100,000$$

$$D_0 = \$2,900,000$$

Now, applying the growing perpetuity equation, we find:

$$PV_{\text{Dividends}} = C_1 / (R - g)$$

$$PV_{\text{Dividends}} = \$2,900,000(1 + .05) / (.15 - .05)$$

$$PV_{\text{Dividends}} = \$30,450,000$$

This is the same answer we found previously. The price per share of stock is the total value of the company's stock divided by the shares outstanding, or:

$$P = \text{Value of all stock} / \text{Shares outstanding}$$

$$P = \$30,450,000 / 1,000,000$$

$$P = \$30.45$$

- b. The value of a share of stock in a company is the present value of its current operations, plus the present value of growth opportunities. To find the present value of the growth opportunities, we need to discount the cash outlay in Year 1 back to the present, and find the value today of the increase in earnings. The increase in earnings is a perpetuity, which we must discount back to today. So, the value of the growth opportunity is:

$$\begin{aligned}\text{NPVGO} &= C_0 + C_1 / (1 + R) + (C_2 / R) / (1 + R) \\ \text{NPVGO} &= -\$22,000,000 - \$8,000,000 / (1 + .15) + (\$7,000,000 / .15) / (1 + .15) \\ \text{NPVGO} &= \$11,623,188.41\end{aligned}$$

To find the value of the growth opportunity on a per share basis, we must divide this amount by the number of shares outstanding, which gives us:

$$\begin{aligned}\text{NPVGO}_{\text{Per share}} &= \$11,623,188.41 / \$1,000,000 \\ \text{NPVGO}_{\text{Per share}} &= \$11.62\end{aligned}$$

The stock price will increase by \$11.62 per share. The new stock price will be:

$$\begin{aligned}\text{New stock price} &= \$30.45 + 11.62 \\ \text{New stock price} &= \$42.07\end{aligned}$$

28. a. If the company continues its current operations, it will not grow, so we can value the company as a cash cow. The total value of the company as a cash cow is the present value of the future earnings, which are a perpetuity, so:

$$\begin{aligned}\text{Cash cow value of company} &= C / R \\ \text{Cash cow value of company} &= \$85,000,000 / .12 \\ \text{Cash cow value of company} &= \$708,333,333.33\end{aligned}$$

The value per share is the total value of the company divided by the shares outstanding, so:

$$\begin{aligned}\text{Share price} &= \$708,333,333.33 / 20,000,000 \\ \text{Share price} &= \$35.42\end{aligned}$$

- b. To find the value of the investment, we need to find the NPV of the growth opportunities. The initial cash flow occurs today, so it does not need to be discounted. The earnings growth is a perpetuity. Using the present value of a perpetuity equation will give us the value of the earnings growth one period from today, so we need to discount this back to today. The NPVGO of the investment opportunity is:

$$\begin{aligned}\text{NPVGO} &= C_0 + C_1 / (1 + R) + (C_2 / R) / (1 + R) \\ \text{NPVGO} &= -\$18,000,000 - 7,000,000 / (1 + .12) + (\$11,000,000 / .12) / (1 + .12) \\ \text{NPVGO} &= \$57,595,238.10\end{aligned}$$

- c. The price of a share of stock is the cash cow value plus the NPVGO. We have already calculated the NPVGO for the entire project, so we need to find the NPVGO on a per share basis. The NPVGO on a per share basis is the NPVGO of the project divided by the shares outstanding, which is:

$$\text{NPVGO per share} = \$57,595,238.10 / 20,000,000$$

NPVGO per share = \$2.88

This means the per share stock price if the company undertakes the project is:

Share price = Cash cow price + NPVGO per share

Share price = \$35.42 + 2.88

Share price = \$38.30

29. a. If the company does not make any new investments, the stock price will be the present value of the constant perpetual dividends. In this case, all earnings are paid as dividends, so, applying the perpetuity equation, we get:

$P = \text{Dividend} / R$

$P = \$7 / .11$

$P = \$63.64$

- b. The investment occurs every year in the growth opportunity, so the opportunity is a growing perpetuity. So, we first need to find the growth rate. The growth rate is:

$g = \text{Retention Ratio} \times \text{Return on Retained Earnings}$

$g = 0.30 \times 0.20$

$g = 0.06$ or 6%

Next, we need to calculate the NPV of the investment. During year 3, 30 percent of the earnings will be reinvested. Therefore, \$2.10 is invested ($\$7 \times .30$). One year later, the shareholders receive a 20 percent return on the investment, or \$0.42 ($\$2.10 \times .20$), in perpetuity. The perpetuity formula values that stream as of year 3. Since the investment opportunity will continue indefinitely and grows at 6 percent, apply the growing perpetuity formula to calculate the NPV of the investment as of year 2. Discount that value back two years to today.

$$\begin{aligned}\text{NPVGO} &= [(\text{Investment} + \text{Return} / R) / (R - g)] / (1 + R)^2 \\ \text{NPVGO} &= [(-\$2.10 + \$0.42 / .11) / (0.11 - 0.06)] / (1.11)^2 \\ \text{NPVGO} &= \$27.89\end{aligned}$$

The value of the stock is the PV of the firm without making the investment plus the NPV of the investment, or:

$P = \text{PV}(\text{EPS}) + \text{NPVGO}$

$P = \$63.64 + \27.89

$P = \$91.53$

Challenge

30. We are asked to find the dividend yield and capital gains yield for each of the stocks. All of the stocks have a 20 percent required return, which is the sum of the dividend yield and the capital gains yield. To find the components of the total return, we need to find the stock price for each stock. Using this stock price and the dividend, we can calculate the dividend yield. The capital gains yield for the stock will be the total return (required return) minus the dividend yield.

$$W: P_0 = D_0(1 + g) / (R - g) = \$4.50(1.10) / (.20 - .10) = \$49.50$$

$$\text{Dividend yield} = D_1/P_0 = 4.50(1.10)/\$49.50 = .10 \text{ or } 10\%$$

$$\text{Capital gains yield} = .20 - .10 = .10 \text{ or } 10\%$$

$$X: P_0 = D_0(1 + g) / (R - g) = \$4.50 / (.20 - 0) = \$22.50$$

$$\text{Dividend yield} = D_1/P_0 = \$4.50/\$22.50 = .20 \text{ or } 20\%$$

$$\text{Capital gains yield} = .20 - .20 = 0\%$$

$$Y: P_0 = D_0(1 + g) / (R - g) = \$4.50(1 - .05) / (.20 + .05) = \$17.10$$

$$\text{Dividend yield} = D_1/P_0 = \$4.50(0.95)/\$17.10 = .25 \text{ or } 25\%$$

$$\text{Capital gains yield} = .20 - .25 = -.05 \text{ or } -5\%$$

$$Z: P_2 = D_2(1 + g) / (R - g) = D_0(1 + g_1)^2(1 + g_2) / (R - g_2) = \$4.50(1.30)^2(1.08) / (.20 - .08) \\ P_2 = \$68.45$$

$$P_0 = \$4.50(1.30) / (1.20) + \$4.50(1.30)^2 / (1.20)^2 + \$68.45 / (1.20)^2 \\ P_0 = \$57.69$$

$$\text{Dividend yield} = D_1/P_0 = \$4.50(1.30)/\$57.69 = .1014 \text{ or } 10.14\%$$

$$\text{Capital gains yield} = .20 - .1014 = .0986 \text{ or } 9.86\%$$

In all cases, the required return is 20 percent, but the return is distributed differently between current income and capital gains. High-growth stocks have an appreciable capital gains component but a relatively small current income yield; conversely, mature, negative-growth stocks provide a high current income but also price depreciation over time.

31. a. Using the constant growth model, the price of the stock paying annual dividends will be:

$$P_0 = D_0(1 + g) / (R - g) = \$3.60(1.05) / (.14 - .05) = \$42.00$$

- b. If the company pays quarterly dividends instead of annual dividends, the quarterly dividend will be one-fourth of annual dividend, or:

$$\text{Quarterly dividend: } \$3.60(1.05)/4 = \$0.9450$$

To find the equivalent annual dividend, we must assume that the quarterly dividends are reinvested at the required return. We can then use this interest rate to find the equivalent annual dividend. In other words, when we receive the quarterly dividend, we reinvest it at the required return on the stock. So, the effective quarterly rate is:

$$\text{Effective quarterly rate: } 1.14^{.25} - 1 = .0333$$

The effective annual dividend will be the FVA of the quarterly dividend payments at the effective quarterly required return. In this case, the effective annual dividend will be:

$$\text{Effective } D_1 = \$0.9450(\text{FVIFA}_{3.33\%,4}) = \$3.97$$

Now, we can use the constant growth model to find the current stock price as:

$$P_0 = \$3.97 / (.14 - .05) = \$44.14$$

Note that we cannot simply find the quarterly effective required return and growth rate to find the value of the stock. This would assume the dividends increased each quarter, not each year.

32. a. If the company does not make any new investments, the stock price will be the present value of the constant perpetual dividends. In this case, all earnings are paid as dividends, so, applying the perpetuity equation, we get:

$$P = \text{Dividend} / R$$

$$P = \$6.25 / .13$$

$$P = \$48.08$$

- b. The investment occurs every year in the growth opportunity, so the opportunity is a growing perpetuity. So, we first need to find the growth rate. The growth rate is:

$$g = \text{Retention Ratio} \times \text{Return on Retained Earnings}$$

$$g = 0.20 \times 0.11$$

$$g = 0.022 \text{ or } 2.20\%$$

Next, we need to calculate the NPV of the investment. During year 3, 20 percent of the earnings will be reinvested. Therefore, \$1.25 is invested (\$6.25 $\times$.20). One year later, the shareholders receive an 11 percent return on the investment, or \$0.138 (\$1.25 $\times$.11), in perpetuity. The perpetuity formula values that stream as of year 3. Since the investment opportunity will continue indefinitely and grows at 2.2 percent, apply the growing perpetuity formula to calculate the NPV of the investment as of year 2. Discount that value back two years to today.

$$\text{NPVGO} = [(\text{Investment} + \text{Return} / R) / (R - g)] / (1 + R)^2$$

$$\text{NPVGO} = [(-\$1.25 + \$0.138 / .13) / (0.13 - 0.022)] / (1.13)^2$$

$$\text{NPVGO} = -\$1.39$$

The value of the stock is the PV of the firm without making the investment plus the NPV of the investment, or:

$$P = PV(\text{EPS}) + \text{NPVGO}$$

$$P = \$48.08 - 1.39$$

$$P = \$46.68$$

- c. Zero percent! There is no retention ratio which would make the project profitable for the company. If the company retains more earnings, the growth rate of the earnings on the investment will increase, but the project will still not be profitable. Since the return of the project is less than the required return on the company stock, the project is never worthwhile. In fact, the more the company retains and invests in the project, the less valuable the stock becomes.
33. Here we have a stock with differential growth, but the dividend growth changes every year for the first four years. We can find the price of the stock in Year 3 since the dividend growth rate is constant after the third dividend. The price of the stock in Year 3 will be the dividend in Year 4, divided by the required return minus the constant dividend growth rate. So, the price in Year 3 will be:

$$P_3 = \$4.20(1.20)(1.15)(1.10)(1.05) / (.12 - .05) = \$95.63$$

The price of the stock today will be the PV of the first three dividends, plus the PV of the stock price in Year 3, so:

$$P_0 = \$4.20(1.20)/(1.12) + \$4.20(1.20)(1.15)/1.12^2 + \$4.20(1.20)(1.15)(1.10)/1.12^3 + \$95.63/1.12^3$$

$$P_0 = \$81.73$$

34. Here we want to find the required return that makes the PV of the dividends equal to the current stock price. The equation for the stock price is:

$$P = \$4.20(1.20)/(1 + R) + \$4.20(1.20)(1.15)/(1 + R)^2 + \$4.20(1.20)(1.15)(1.10)/(1 + R)^3 + [\$4.20(1.20)(1.15)(1.10)(1.05)/(R - .05)]/(1 + R)^3 = \$98.65$$

We need to find the roots of this equation. Using spreadsheet, trial and error, or a calculator with a root solving function, we find that:

$$R = .1081 \text{ or } 10.81\%$$

35. In this problem, growth is occurring from two different sources: The learning curve and the new project. We need to separately compute the value from the two different sources. First, we will compute the value from the learning curve, which will increase at 5 percent. All earnings are paid out as dividends, so we find the earnings per share are:

$$\text{EPS} = \text{Earnings} / \text{total number of outstanding shares}$$

$$\text{EPS} = (\$15,000,000 \times 1.05) / 10,000,000$$

$$\text{EPS} = \$1.58$$

From the NPVGO mode:

$$P = E/(R - g) + \text{NPVGO}$$

$$P = \$1.58/(0.10 - 0.05) + \text{NPVGO}$$

$$P = \$31.50 + \text{NPVGO}$$

Now we can compute the NPVGO of the new project to be launched two years from now. The earnings per share two years from now will be:

$$\text{EPS}_2 = \$1.58(1 + .05)^2$$

$$\text{EPS}_2 = \$1.6538$$

Therefore, the initial investment in the new project will be:

$$\text{Initial investment} = .30(\$1.6538)$$

$$\text{Initial investment} = \$0.50$$

The earnings per share of the new project is a perpetuity, with an annual cash flow of:

$$\text{Increased EPS from project} = \$6,500,000 / 10,000,000 \text{ shares}$$

$$\text{Increased EPS from project} = \$0.65$$

So, the value of all future earnings in year 2, one year before the company realizes the earnings, is:

$$\text{PV} = \$0.65 / .10$$

$$\text{PV} = \$6.50$$

Now, we can find the NPVGO per share of the investment opportunity in year 2, which will be:

$$\text{NPVGO}_2 = -\$0.50 + 6.50$$

$$\text{NPVGO}_2 = \$6.00$$

The value of the NPVGO today will be:

$$\text{NPVGO} = \$6.00 / (1 + .10)^2$$

$$\text{NPVGO} = \$4.96$$

Plugging in the NPVGO model we get;

$$P = \$31.50 + 4.96$$

$$P = \$36.46$$

Note that you could also value the company and the project with the values given, and then divide the final answer by the shares outstanding. The final answer would be the same.